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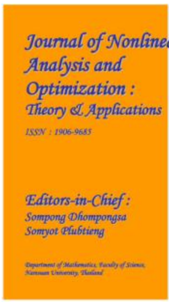

A STUDY OF FIBONACCI NUMBERS AND THE GOLDEN RATIO

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Abstract

A Fibonacci sequence is a sequence in which each number is the sum of the two proceeding ones. The Fibonacci sequence can be applied to finance by using four techniques including retracements, arcs. Fans and time zones. The golden ratio is also known as the golden number, golden proportion, is a ratio between two numbers that equals approximately 1.618. In this paper we have to describe about Fibonacci number and its Properties. Fibonacci number under modular representation and about Pascal Triangle and also about Binet's Formula

Key Words: Fibonacci number, Fibonacci sequence**,** Pascal's Triangle.

Introduction

The term "Fibonacci numbers" is used to describe the series of number generate by the patterns

1,1,2,3,5,8,13,21,34,55,89,144,……..

Where each number in the sequence is given by the sum of the previous two terms.

This pattern is given by $f_{n=1}$, $f_{2=1}$ and the recursive formula

$$f_n = f_{n-1} + f_{n-2}, \quad n>2.$$

First derived from the famous "rabbit problem" of 1228, the Fibonacci numbers were originally used to represent the number of pairs of rabbits born of one pair in a certain population. Let us assume that a pair of rabbits is introduced into a certain place in the first month of the year. This pair of rabbits will produced one pair of offspring every month, and every pair of rabbits will begin to reproduce exactly two months after being born. No rabbit ever dies, and every pair of rabbits will reproduce on schedule.

So, in the first month, we have only the first pair of rabbits. Likewise, in the second month, we again have only our initial pair of rabbits. However, by the third month, the pair will give birth to another pair of rabbits, and there will now be two pairs. Continuing, on we find that in month four we will have 3 pairs, then 5 pairs in month five, then 8,13,21,34,…….,etc, continuing in this manner. It is quite apparent that this sequence directly corresponds with the Fibonacci sequence introduced above, and indeed, this is the first problem ever associated with the now-famous numbers.

Now that we have seen one application of the Fibonacci numbers and established a basic definition, we will go on to examine some of the simple properties regarding the Fibonacci numbers and their sums.

Properties of The Fibonacci Numbers

To begin our research on the Fibonacci sequence, we will first examine some simple, yet important properties regarding the Fibonacci numbers. These properties should help to act as a foundation upon which we can base future research and proofs.

The following properties of Fibonacci numbers were proved in the book Fibonacci numbers by N.N. Vorob' ev.

Lemma 1.

The sum of the first n Fibonacci number can be expressed as

$u_1+u_2+\ldots\ldots\ldots+u_{n-1}+u_n=u_{n+2}-1.$

Proof:

From the definition of the Fibonacci sequence, we know

$u_1=u_3-u_2,$

$u_2=u_4-u_3,$

$u_3=u_5-u_4,$

$\ldots\ldots\ldots$

$u_{n-1}=u_{n+1}-u_{n+2},$

$u_n=u_{n+2}-u_{n+1}.$

We now add these equations to find

$u_{1+}u_2+\ldots\ldots\ldots+u_{n-1}+u_n=u_{n+2}-u_2.$

Recalling that u_2=1, we see this equation is equivalent to our initial conjecture of

$u_1+u_2+\ldots\ldots\ldots+u_{n-1}+u_n=u_{n+2}-1.$

Lemma 2.

The sum of the odd terms of the Fibonacci sequence

$u_1+u_2+\ldots\ldots+u_{2n-1}=u_{2n}.$

Proof:

Again looking at individual terms, we see from the definition of the sequence that

$u_1=u_2,$

$u_3=u_4-u_2$

$u_5=u_6-u_4,$

$\ldots\ldots\ldots$

$u_{2n-1}=u_{2n}-u_{2n-2}.$

If we now add these equation term by term, we are left with required result from above

Fibonacci Numbers And Pascal's Triangle.

Pascal's Triangle

The Pascal's triangle .The Fibonacci numbers can be derived by summing of element on the rising diagonal lines in the Pascal's triangle . In similar, we will show that Fibonacci p-numbers can be read from the Pascal's triangle.

The Fibonacci numbers share an interesting connection with the triangle of binomial coefficients known as Pascal's triangle.

Pascal's triangle typically takes the form:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
.....

In this depiction we have oriented the triangle to the left for ease of use in our future application. Pascal's triangle, as may already be apparent, is a triangle in which the topmost entry is 1 and each following entry is equivalent to the term directly above plus the term above and to the left.

Another representation of Pascal's triangle takes the form:

c_o^0

$c_1^o \quad c_1^1$

$c_2^0 \quad c_2^1 \quad c_2^2$

$c_3^0 \quad c_3^1 \quad c_3^2 \quad c_3^3$

$c_4^0 \quad c_4^1 \quad c_4^2 \quad c_4^3 \quad c_4^4$

In this version of Pascal's triangle , we have $c_j^i = \frac{k!}{i!(k-i)!}$, where I the column and k represents the row the given term is in . Obviously, we have designated the first row as row 0 and the first column as column 0.

Finally, we will now depict Pascal's triangle with its rising diagonals.

The diagonal lines drawn through the number of this triangle are called the "rising diagonals' of Pascal's triangle. So, for example , the lines passing through 1,3,1 or1,4,3 would both indicate different rising diagonal to the Fibonacci numbers.

The Golden Ratio.

In calculating the ratio of two successive Fibonacci numbers, $\frac{u_{n+1}}{u_n}$, we find that as n increases bound, ratio approaches $\frac{1+\sqrt{5}}{2}$.

Theorem 1.

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{1+\sqrt{5}}{2}$$

Proof:

Since $u_{n+1} = u_n + u_{n-1}$,

By definition , it follows that

$$\frac{u_{n+1}}{u_n} = 1 + \frac{u_{n-1}}{u_n}.$$

Now, let $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = L$

We then see that

$$\lim_{n\to\infty} \frac{u_{n-1}}{u_n} = \frac{1}{L}$$

We now have the statement

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = 1 + \lim_{n\to\infty} \frac{u_{n-1}}{u_n}$$

Which is equivalent to the equation $L = 1 + \frac{1}{L}$.

This equation can then be written as $L^2 - L - 1 = 0$

Which is easily solved using the quadratic formula. By using the quadratic formula, we have

$$L = \frac{1 \pm \sqrt{5}}{2}$$

Thus , we arrive at our desired result of

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{1+\sqrt{5}}{2}$$

Even for relatively low values of n, this ratio produces a very small error. For example

$$\frac{u_{11}}{u_{10}} = \frac{89}{55} \approx 1.6182, \quad \text{and}$$

$$\frac{1+\sqrt{5}}{2} \approx 1.6180.$$

The value $\frac{1+\sqrt{5}}{2}$ is the positive root of the equation $x^2 - x^1 - 1 = 0$ and is often referred to as $\propto$. It arises often enough in mathematics and has such interesting properties that we also frequently

BINET'S FORMULA

A generating function is a function in which the coefficients of a power series give the answers to a counting problem.

Fibonacci Numbers Under modular Representation

Introduction to Modular Representation. We will now examine the Fi- bonacci numbers under modular addition.

First, we will familiarize ourselves with modulo notation. Given the integers *a*, *b* and *m*, the expression *a* *b*(*mod m*) (pronounced "a is congruent to b modulo n") means that *a* *b* is a multiple of *m*. For 0 *a* < *n*, the value *a* is equivalent to the remainder, or residue, of *b* upon division by *n*.

So, for example

$3 \equiv 13(mod\ 10)$ or $2 \equiv 17(mod\ 5)$.

It is also convenient to note that

$$a(mod\ m) + b(mod\ m) = (a + b)(mod\ m).$$

Subtraction and multiplication work similarly.

Now that we are comfortable with basic modular operations, we shall examine an example of the first 12 Fibonacci numbers (mod 2).

Example . Fibonacci numbers:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144...

Fibonacci numbers (mod 2):

1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0...

It should be apparent that only the pattern of 1, 1, 0 repeats throughout the Fibonacci series (*mod* 2). So, we can say that the series is periodic, with the period being 3 in this case, since there is a repetition of only three terms. We will later go on to prove that all modular representaions of the Fibonacci numbers are periodic. Furthermore, we will show that this period is solely determined by the two numbers directly following the first 0 within the series.

The Fibonacci Numbers Modulo m. Before attempting to prove any ma- jor conclusions about the Fibonacci numbers modulo *m*, it may help us to first examine the Fibonacci series for many values of *m*. Let us look at the first 30 terms of the Fibonacci series (*mod m*), where *m* ranges from 2 to 10. First of all, the first 30 Fibonacci numbers are:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832.

So, by using Maple, we find

$F\ (mod\ 2)$ = 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0
$F\ (mod\ 3)$ = 1, 1, 2, 0, 2, 2, 1, 0, 1, 1, 2, 0, 2, 2, 1, 0, 1, 1, 2, 0, 2, 2, 1, 0, 1, 1, 2, 0, 2, 2
$F\ (mod\ 4)$ = 1, 1, 2, 3, 1, 0, 1, 1, 2, 3, 1, 0, 1, 1, 2, 3, 1, 0, 1, 1, 2, 3, 1, 0, 1, 1, 2, 3, 1, 0
$F\ (mod\ 5)$ = 1, 1, 2, 3, 0, 3, 3, 1, 4, 0, 4, 4, 3, 2, 0, 2, 2, 4, 1, 0, 1, 1, 2, 3, 0, 3, 3, 1, 4, 0
$F\ (mod\ 6)$ = 1, 1, 2, 3, 5, 2, 1, 3, 4, 1, 5, 0, 5, 5, 4, 3, 1, 4, 5, 3, 2, 5, 1, 0, 1, 1, 2, 3, 5, 2
$F\ (mod\ 7)$ = 1, 1, 2, 3, 5, 1, 6, 0, 6, 6, 5, 4, 2, 6, 1, 0, 1, 1, 2, 3, 5, 1, 6, 0, 6, 6, 5, 4, 2, 6
$F\ (mod\ 8)$ = 1, 1, 2, 3, 5, 0, 5, 5, 2, 7, 1, 0, 1, 1, 2, 3, 5, 0, 5, 5, 2, 7, 1, 0, 1, 1, 2, 3, 5, 0
$F\ (mod\ 9)$ = 1, 1, 2, 3, 5, 8, 4, 3, 7, 1, 8, 0, 8, 8, 7, 6, 4, 1, 5, 6, 2, 8, 1, 0, 1, 1, 2, 3, 5, 8
$F\ (mod\ 10)$ = 1, 1, 2, 3, 5, 8, 3, 1, 4, 5, 9, 4, 3, 7, 0, The Period of F(mod m).
7, 7, 4, 1, 5, 6, 1, 7, 8, 5, 3, 8, 1, 9,

Lemma. The Fibonacci series under modular representation is always periodic.

Proof. Since the Fibonacci series is recursive, we know that any pair of consecutive terms will completely determine the rest of the series. Furthermore, under modular representation, we know that each Fibonacci number will be represented as some residue $0 \leq F\ (mod\ m) < m$. Thus, there are only m possible values for any given $F\ (mod\ m)$ and hence $m\ m = m^2$ possible pairs of consecutive terms within the sequence. Since m^2 is finite, we know that some pair of terms must eventually repeat itself. Also, as any pair of terms in the Fibonacci sequence determines the rest of the sequence, we see that the Fibonacci series modulo m must repeat itself at some point, and thus must be periodic. Q

Now, we will let $k(m)$ denote the period of $F\ (mod\ m)$. That is, $k(m)$ represents the number of terms of $F\ (mod\ m)$ before the cycle starts to repeat again.

So, analyzing the above data, we can determine the period of all but the last series.

$k(2) = 3$, $k(3) = 8$, $k(4) = 6$, $k(5) = 20$, $k(6) = 24$, $k(7) = 16$, $k(8) = 12$, $k(9) = 24$.

While it appears there may be some connection between these values, it may be more convenient to analyze a larger sample size. To attain this larger sample size, we will make use of a list of periods of $F\ (mod\ m)$ given by Marc Renault, associate.

Conclusion:

In this paper we discussed about Fibonacci series and Fibonacci number and its properties. Here we described about Fibonacci number under modular representation. This is the series of numbers represents a fundamental mathematical pattern present in many natural phenomena. Moreover the Fibonaci squene has a practical applications in various fields, such as stock market analysis and population growth modeling.

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